Estimates of Uncertainty of the Calibration of Balances

Chart 1 shows the uncertainty (estimates) of the calibration of representative electronic balances.

Chart 1. Uncertainty of calibration (estimates)

Minimum weighing value	Model	Calibration point (No tare)	Extended uncertainty (k=2)
0.001 mg	BM-20	1 g	0.019 mg
0.01 mg	GH-202	50 g	0.17 mg
	GH-252	100 g	0.31 mg
	BM-252	100 g	0.29 mg
0.1 mg	GR-120/GH-120	100 g	0.4 mg
	GR-200/GH-200/BM-200	200 g	0.6 mg
	BM-500	500 g	1.6 mg
0.001 g	GX-200/GF-200	200 g	0.004 g
	GX-600/GF-600	500 g	0.004 g
	GX-1000/GF-1000	1 kg	0.005 g
0.01 g	GX-2000/GF-2000	2 kg	0.04 g
	GX-6100/GF-6100	5 kg	0.05 g

Note:
1. The uncertainties above are premised on normal operation of the electronic balance under good conditions.
2. For the components of calibration uncertainty, the values below are applied as 1. Calibration weight, 2. Repeatability, 3. Rounding error, 4. Eccentricity and 5. Temperature characteristics.
   Note that while calibration uncertainty usually includes "accuracy" (the deviation of calibration results), the uncertainty estimates above do not take into account components of "accuracy" because "accuracy" is influenced by the accuracy of the calibration weight used in sensitivity adjustment. Similarly, aging over time of the calibration weight, the atmospheric pressure during calibration of the calibration weight, and pressure changes during balance calibration also depend on the management of the calibration weight so they are not included as components of uncertainty.
3. Uncertainty is influenced by the condition of the electronic balance and the environment during calibration. To be exact, it is necessary to estimate the uncertainty of the actual instrument under the calibration environment. Therefore, the calibration uncertainties above are examples of calibration uncertainty estimated using the actual instrument and are not guaranteed values.

1. Uncertainty due to the weight used for calibration

Extended uncertainty (coverage factor k=2) for the calibration weights to be used are listed in the chart below.

Nominal value	Extended uncertainty (k=2)
1 g	0.018 mg
50 g	0.101 mg
100 g	0.15 mg
200 g	0.25 mg
500 g	1.0 mg
1 kg	1.5 mg
2 kg	3 mg
5 kg	5 mg

The extended uncertainties for the calibration weights above indicate the measurement performance when calibrating an OIML Class E2 weight in A&D's calibration laboratory.

2. Uncertainty due to repeatability
   Use the catalog specifications (reproducibility) of each balance.
3. Uncertainty due to rounding error
   Calculate from the minimum display value of the relevant balance.
4. Uncertainty due to eccentricity
   Calculate from the specifications at the time the relevant balance was manufactured.
5. Uncertainty due to temperature characteristics
   Calculate based on the catalog specifications (sensitivity drift) of the relevant balance with the temperature change during calibration set within $\Delta 2$°C.

Example of Calculating the Uncertainty of Balance Calibration

1. Introduction

In this example, the actual "uncertainty of balance calibration" for a GR-200 is calculated.

GR-200 specifications

Capacity	:*Max*	210 g
Minimum weighing value (scale interval)	:*d*	0.1 mg

The components of uncertainty during calibration include the following items.

1-1 Uncertainty due to the calibration weight

1-2 Uncertainty due to the calibration work

1. Repeatability
2. Eccentricity
3. Accuracy

1-3 Other causes of uncertainty

1. Temperature characteristics
2. Rounding error

2. Preparation

While referring to the operation manual, set up the balance in an appropriate location, and then warm up the balance for the specified time. (Leave the power on.)

3. Sensitivity adjustment

Adjust the sensitivity before calibration if necessary. If the sensitivity is adjusted, note this fact in the calibration results as a condition of calibration.

4. Repeatability

As an example, the following procedure shows how to measure repeatability with a test load of 200 g, which is equal to or greater than 0.1*Max* (Capacity × 0.1).

Measurement is performed six times.

1. Reset the balance to zero, and then confirm that the indicator value is zero.
2. Load a weight equivalent to the test load on the center of the pan, and then record the indicator value.
3. Remove the weight.

Repeat steps 1 to 3 six times.

Example measurement results

Measurement order i	1	2	3	4	5	6
Indicator value I_i	200.0000 g	200.0000 g	200.0001 g	200.0000 g	200.0001 g	200.0001 g
Average value $\bar{I}$	200.00005 g					
$I_i - \bar{I}$	-0.05 mg	-0.05 mg	0.05 mg	-0.05 mg	0.05 mg	0.05 mg

From these results, use the following formula to calculate the dispersion (Vr) due to repeatability.

$$V\mathrm{r} = S^2 = \frac{1}{\mathrm{n}-1}\sum_{\mathrm{i}=1}^{\mathrm{n}}(I\mathrm{i}-\bar{I})^2$$

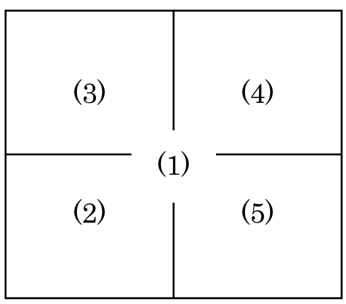
$$= 3.0000\times10^{-3}\ \mathrm{mg}^2 \quad \dots (1)$$

Here, $\bar{I}$ is the average of the indicator values

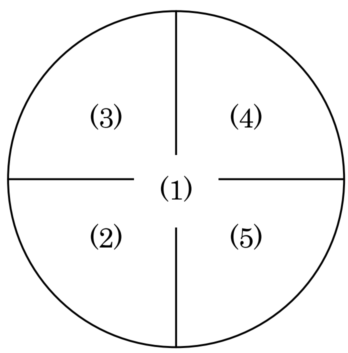
5. Eccentricity

As an example, the following procedure shows how to measure eccentricity of a test load (P) of a 100 g, which is equal to or greater than 0.3Max.

1. Before measurement, reset the balance to zero, and then confirm that the indicator value is zero.
2. Load a 100 g weight, which is equivalent to the test load, on the center of the pan (position 1), and then record the indicator value I1.
3. Place the same weight at position 2, and then record the indicator value I2.
4. Place the same weight at positions 3, 4, and 5 in turn, while recording the respective indicator values I3, I4, and I5.



For a square weighing pan



For a round weighing pan

Positions 2 to 5 are in the centers of the quarters

Example measurement results

Measurement order i	1	2	3	4	5
Measurement position	(1)	(2)	(3)	(4)	(5)
Indicator value I_i	100.0000 g	99.9999 g	100.0002 g	100.0001 g	99.9998 g
Difference from center		-0.1 mg	0.2 mg	0.1 mg	-0.2 mg

From the results, calculate E1 (the eccentricity standardized to a test load of $Max/3$) from the difference ΔE(the maximum difference for center loading and eccentric loading).

$$E1=\Delta E\cdot \frac{Max}{3P}$$

If Max (capacity) = 210 g, P (test load) = 100 g, and ΔE = 0.2 mg (from the eccentricity measurement above) are substituted, the following value results.

$E1$=0.2 mg × 210 g/300 g=0.14 mg

Ve (the relative dispersion of the eccentricity) is presumed to have a uniform distribution with a maximum of ΔE ($\pm\Delta E$).

$$V\mathrm{e} = U\mathrm{e}^2 = (\frac{1}{\sqrt{3}})^2 \times (\frac{E1}{Max})^2 = \frac{1}{3} \times (\frac{0.14}{210000})^2$$

$$= 1.4815 \times 10^{-13} \quad \ldots (2)$$

6. Accuracy

As an example, accuracy is measured using test loads, including those with and without tare loads, of the following weights.

Weight type	Use	Nominal value	Conventional mass	Extended uncertainty U(k=2)	Dispersion of uncertainty of calibration weight Vs
$W1$	Calibration	200 g	200 g+0.15 mg	0.25 mg	1.5625×10^{-2}mg^2
$W2$	Calibration	50 g	50 g-0.06 mg	0.101 mg	2.5503×10^{-3}mg^2
$T1$	Tare load	50 g			
$T2$	Tare load	100 g			
$T3$	Tare load	150 g			

Note: It is acceptable for the 150 g for $T3$ to be comprised of $T1$ and $T2$.

Based on coverage factor k=2, calculate Vs (the dispersion of uncertainty of calibration weight) using the following formula.

$V\mathrm{s} = (U/2)^2$

1. Before measurement, reset the balance to zero. Check that the indicator value is zero, load calibration weight $W1$ at the center of the pan, and then record the indicator value $I1$. Next, remove the weight.
2. As in step 1, load calibration weight $W2$, and then record indicator value $I2$.
3. Load weight $T1$ as the tare load weight, and then take tare. Check that the indicator value is zero, place calibration weight W2 at the center of the pan, and then record the indicator value $I3$. Next, remove the weight.
4. As with Step 3, load calibration weight $T2$ as the tare load, and then record indicator value $I4$ for

calibration weight $W2$.

5. As with Step 3, load calibration weight $T3$ as the tare load, and then record indicator value $I5$ for calibration weight $W2$.

Example test loads and measurement results

Measurement order i	1	2	3	4	5
Tare load	0 g	0 g	50 g	100 g	150 g
Weight for tare	-	-	*T1*	*T2*	*T3*
Calibration weight	*W1*	*W2*	*W2*	*W2*	*W2*
Calibration weight value	200.00015 g	49.99994 g	49.99994 g	49.99994 g	49.99994 g
Indicator value I_i	**200.0000 g**	**49.9998 g**	**49.9999 g**	**50.0000 g**	**49.9999 g**
Deviation	**-0.15 mg**	**-0.14 mg**	**-0.04 mg**	**0.06 mg**	**-0.04 mg**

7. Temperature characteristics

Establish the temperature change (ΔT) during balance calibration as follows. For the temperature range, a uniform distribution with a maximum of $\frac{1}{2}\Delta T$ is assumed. For the temperature characteristics value TK (sensitivity drift), calculate the relative dispersion (Vt) of the temperature characteristics using the following formula based on the specifications of the balance.

ΔT=2°C … Establish the temperature change during balance calibration as 2°C.

TK=2ppm/°C … Balance specification

$$V\mathrm{t} = U\mathrm{t}^2 = \left(\frac{\Delta T}{2\sqrt{3}} \times TK\right)^2 = \frac{1}{12}(\Delta T \times TK)^2$$

$$= \frac{1}{12}\left(2^\circ\mathrm{C} \times \frac{2\mathrm{ppm}}{^\circ\mathrm{C}}\right)^2 = 1.3333 \times 10^{-12} \quad \ldots (3)$$

8. Rounding error

The uncertainty for the resolution (scale interval d) of the display is assumed to be the uniform distribution with a maximum of $d/2$. This assumption is based on the following formula using the zero point before measurement and the measurement values.

$$V\mathrm{d} = U\mathrm{d}^2 = \left(\frac{d}{2\sqrt{3}}\right)^2 \times 2 = \frac{1}{6} \times d^2$$

$$= \frac{1}{6} \times 0.1\,\mathrm{mg}^2 = 1.6667 \times 10^{-3}\ \mathrm{mg}^2 \quad \ldots (4)$$

9. Extended uncertainty

Use the following formula to calculate the extended uncertainty U (coverage factor k=2) at each measurement point in “6. Accuracy”.

$$U = k \times \sqrt{[V\mathrm{r} + V\mathrm{d} + V\mathrm{s} + V\mathrm{e} \times W\mathrm{i}^2 + V\mathrm{t} \times W\mathrm{i}^2]} = k \times \sqrt{V}$$

The items to substitute are listed below.

- Vr: The repeatability dispersion. Substitute the calculation result (1) of "4. Repeatability".
- Vd: The rounding error dispersion. Substitute the calculation result (4) of "8. Rounding error".
- Vs: The calibration weight uncertainty dispersion. Calculate from the calibration weight uncertainty used in the measurement of "6. Accuracy". (See "6. Accuracy.)
- Ve: The relative dispersion of the eccentricity. Substitute the calculation result of "5. Eccentricity".
- Vt: The relative dispersion of the temperature characteristics. Substitute the calculation results (3) of "7. Temperature characteristics".
- Wi: Calibration weight
- k: Coverage factor (k=2)
- Note: For eccentricity (Ve) and temperature characteristics (Vt), calculate the dispersion with each uncertainty being proportional to weighing value.

		Calibration weight	
		W1(200 g)	W2(50 g)
Repeatability	Vr	3.0000×10^{-3} mg^2	
Rounding error	Vd	1.6667×10^{-3} mg^2	
Calibration weight	Vs	1.5625×10^{-2} mg^2	2.5503×10^{-3} mg^2
Eccentricity	Ve× Wi^2	$1.4815 \times 10^{-13} \times (200000\ \mathrm{mg})^2$ $= 5.9259 \times 10^{-3}$ mg^2	$1.4815 \times 10^{-13} \times (50000\ \mathrm{mg})^2$ $= 3.7037 \times 10^{-4}$ mg^2
Temperature characteristics	Vt× Wi^2	$1.3333 \times 10^{-12} \times (200000\ \mathrm{mg})^2$ $= 5.3333 \times 10^{-2}$ mg^2	$1.3333 \times 10^{-12} \times (50000\ \mathrm{mg})^2$ $= 3.3333 \times 10^{-3}$ mg^2
Dispersion total	V	7.9551×10^{-2} mg^2	1.0921×10^{-2} mg^2
Extended uncertainty (k=2)	U	**0.6 mg**	**0.3 mg**

Note: Calculate the extended uncertainty result by rounding up the last digit of the acquired value.

*For W*1 (200 g)

$$\mathrm{U} = \mathrm{k} \times \sqrt{V} = 2 \times \sqrt{(7.9551 \times 10^{-2}\ \mathrm{mg}^2)}\ = 0.56\ \mathrm{mg} \rightarrow 0.6\ \mathrm{mg}$$

*For W*2 (50 g)

$$\mathrm{U} = \mathrm{k} \times \sqrt{V} = 2 \times \sqrt{(1.0921 \times 10^{-2}\ \mathrm{mg}^2)}\ = 0.21\ \mathrm{mg} \rightarrow 0.3\ \mathrm{mg}$$

10. Calibration results

Based on the results of "6. Accuracy" and "9. Extended uncertainty", the balance calibration results are as follows.

Tare load	Nominal value	Deviation	Extended uncertainty
0 g	200 g	-0.15 mg	0.6 mg
0 g	50 g	-0.14 mg	0.3 mg
50 g	50 g	-0.04 mg	0.3 mg
100 g	50 g	0.06 mg	0.3 mg
150 g	50 g	-0.04 mg	0.3 mg

For the extended uncertainty, the coverage factor is k=2.

References

JCSS Guide for estimating uncertainty

Scope: Mass

Weighing instrument: Balance

(Version 7)

ASG104 Beginner's guide on uncertainty of measurement